Sparse matrix ordering method with a quantum annealing approach and its parameter tuning

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➢ Parameter tuning for Quantum Annealing

- Quantum computer has the potential to solve NP-problems at high speed
- Quantum Annealing (QA) : a kind of realization method of Quantum computers
- Tuning of parameters is essential to solve problems with QA

  → there is no standard on how to set parameters

We discuss need for parameter tuning in QA

Outline

I. Quantum annealing

II. Fill-in reduction ordering for a sparse direct solver

   a. Minimum degree ordering

   b. Quantum Minimum Fill-in ordering

III. Tuning for solving problems using quantum annealing

IV. Experimental Result

   a. Fixed deletion order matrices

   b. Random matrices

V. Conclusion

Outline

Quantum Annealing (QA)

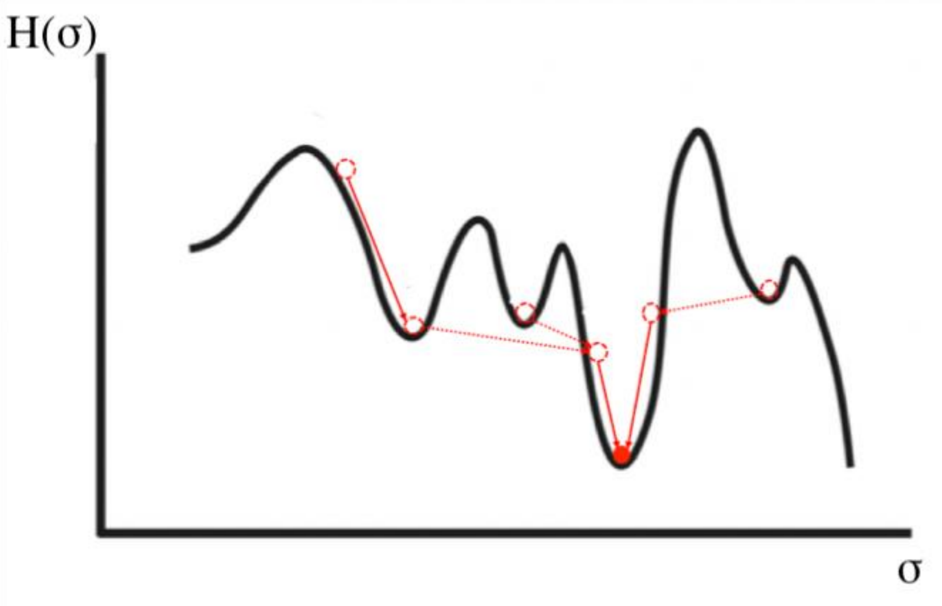
- A kind of realization method for quantum computers
- Search for the location with the minimum energy
- combinatorial optimization problems(COPs) solver

$$H(\sigma) = -\sum_{i<j} J_{ij}\,\sigma_i\sigma_j - \sum_{i=1}^{N} h_i\sigma_i$$

$\boldsymbol{\sigma = \{-1, +1\}}$

J_{ij} : interaction b/w σ_i- σ_j

h : local field of σ_i



$$H(x) = \sum_i \sum_j Q_{ij} x_i x_j$$

$\boldsymbol{x = \{0, 1\}},$

Q_{ij} : interaction b/w x_i- x_j

$$\boldsymbol{x_i = \frac{\sigma_i + 1}{2}}$$

T. Kadowaki and H. Nishimori, "Quantum annealing in the transverse Ising model", Physical Review E, Vol. 58,No. 5, pp. 5355, 1998.
A. Lucas, "Ising formulations of many NP problems", Frontiers in Physics, 2014.

I. Quantum Annealing

➢ Ising machine

- reproduces the behavior of QA
- Solves COPs using meta-heuristics

Table 1. Ising machines

	D-Wave 2000Q	HITACHI CMOS Annealing	Fujitsu Digitar Annealer	Fixstars Amplify AE
Computation method	Quantum annealing	Digital circuit	Digital circuit	GPU
Maximum number of qubits	2,048 (16×16×8)	61,952 (352×176)	1,024/8,192	100,000 or more
Coupling graph	Chimera graph	King graph	All couplings	All couplings
Number of qubits converted to full coupling	64	176	1,024/8,192	65,536

Fixstars, https://quantum.fixstars.com/techresouces/annealing-method/programming/index.html

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II. Fill-in reduction ordering for a sparse direct solver

- **Sparse matrix**
  - A matrix whose elements are mostly zeros
  - It can reduce memory consumption by storing only the nonzero elements of the matrix
  - When calculating with a direct solver, elements that were zero become non-zero (fill-in)

    → increases computation and memory usage
  - Ordering (swaps the rows and columns of the matrix) makes less fill-ins

    → Finding an ordering that minimizes the fill-in is an NP problem, & difficult to compute in reasonable time

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 2 \\ 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 3 & 2 & 0 \\ 2 & 0 & 0 & 0 & 2 \end{bmatrix} x = b$$

➢ Elimination graph

- The nonzero pattern of a symmetric matrix can be transformed into a graph
- the row/column numbers as nodes & the nonzero elements as edges
- The order of node deletion = **ordering**

  added edge = **fill-in**

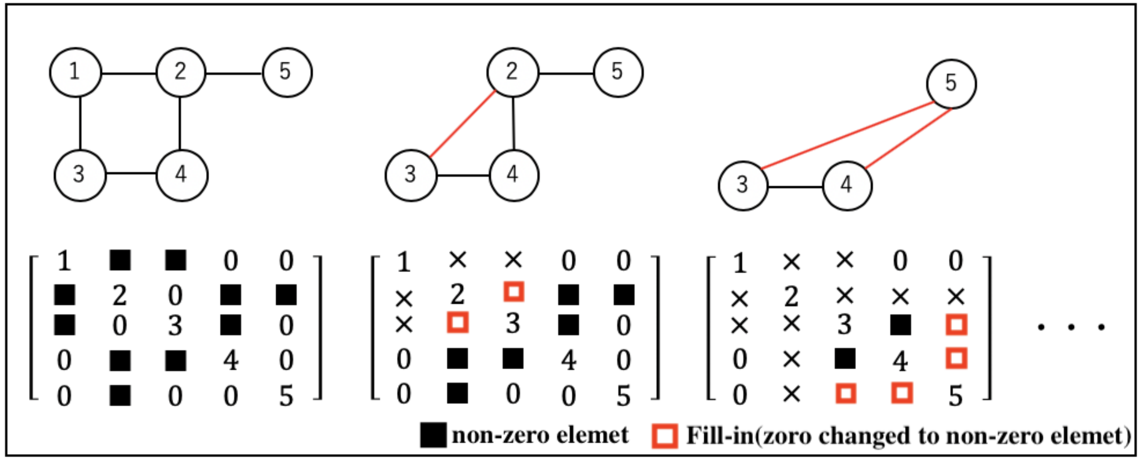

Fig1. Elimination graph

Find the order of vertex deletion that minimizes the edge additions

II.a. Minimum Degree ordering (MD)

- One of the traditional method of graph theoretic ordering for symmetric matrices
- At each step, delete the node with the minimum degree & add the necessary edges
- This strategy has been very successful for problems that aren't huge & for graphs with no cycles

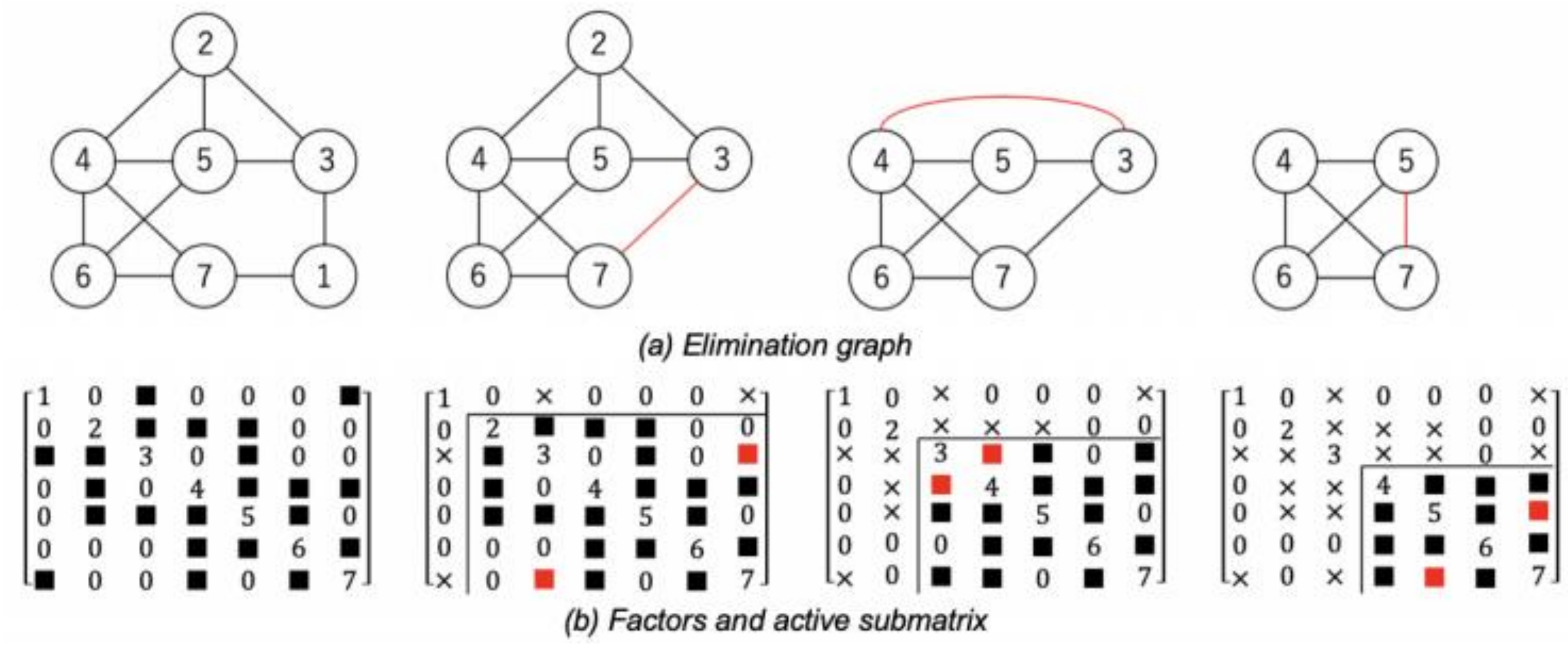

Fig2.　Minimum Degree Ordering

Quantum Minimum Fill-in ordering (QMF)

- Find the deletion order of each node with minimum edge addition using QA
- Qubits are set for the deletion order of each node and for non-existent edges

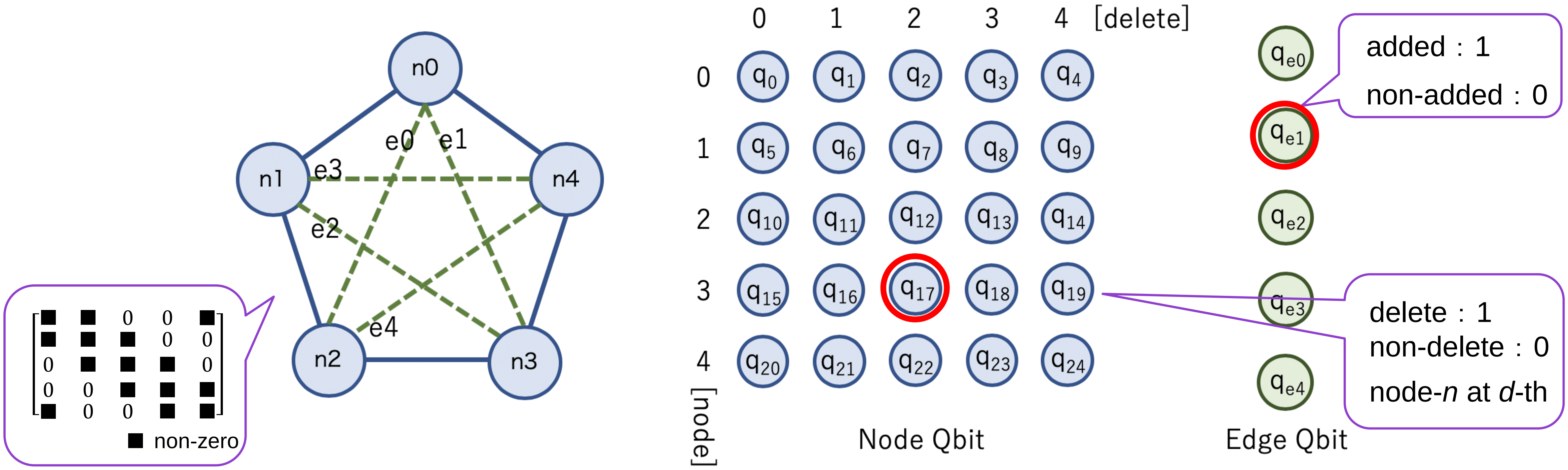

Fig3. Quantum Minimum Fill-in Ordering

II.b. Quantum Minimum Fill-in ordering (QMF)

Hamiltonian of QMF

- Cost function: Minimize the total number of edges added by node deletion

$$H_{cost} = \sum_{edge \in E} x_{edge}$$

- Constraint function 1: Delete all node & don't deleted some nodes at the same time

$$H_{sub1} = \sum_{node=0}^{N-1} \left(\sum_{delete=0}^{N-1} x_{node,delete} - 1 \right)^2 + \sum_{delete=0}^{N-1} \left(\sum_{node=0}^{N-1} x_{node,delete} - 1 \right)^2$$

- Constraint function 2: Add the edge caused by node deletion

$$H_{sub2} = \sum_{edge \in E} \sum_{node \notin i,ii} \sum_{delete=0}^{N-1} \left\{ (1 - x_{edge})\, x_{node,delete} \left(1 - \sum_{j=0}^{delete} x_{i,j}\right)\left(1 - \sum_{j=0}^{delete} x_{ii,j}\right) x_i x_j \right\}^2$$

$$\boldsymbol{H = A * H_{cost} + B * H_{sub1} + C * H_{sub2}}$$

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III. Tuning for solving problems using quantum annealing

a. Weights of the cost and constraint functions in the Hamiltonian
b. Hardware parameters of the Ising machine

III. Tuning for solving problems using quantum annealing

a. Weights of the cost and constraint functions in the Hamiltonian

b. Hardware parameters of the Ising machine

- we give the weights of the cost and constraint functions when we formulate the Hamiltonian
- The ease of finding the optimal solution depends on the weights given to the cost & constraint function
- In general…
  - ✓ Give the cost function a weight that is small enough to satisfy the constraints
  - ✓ If the weight of the constraint function is too large, it’s difficult to obtain a low-cost solution
  - ✓ If there are multiple constraints, set the weights according to the priority

III. Tuning for solving problems using quantum annealing

a. Weights of the cost and constraint functions in the Hamiltonian

b. Hardware parameters of the Ising machine

- In the Hamiltonian of the QMF…
  - ✓ the conflicts related to H_{sub2} exist
  - ✓ it is necessary to set appropriate weights that balance these three functions

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Experiment setup

- Evaluate :

$$optimal\ solution\ rate = \frac{optimal\ soluton}{culculations}$$

- Ising machine : Amplify AE

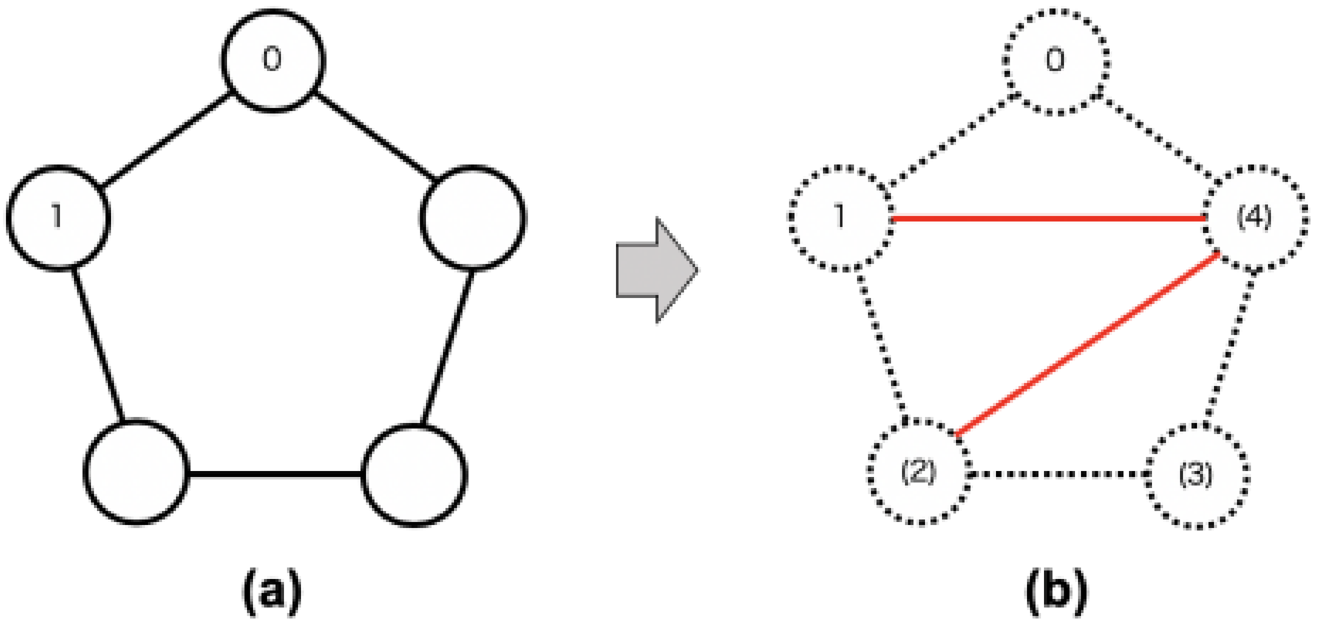
- Problem size & Parameters :

		matrix size	Required qubits
Problem size		5×5	3904
		6×6	11993
		7×7	32268
weight	A	$0.05 - 1.00$	
	B	1.00	
	C	$0.50 - 1.00$	

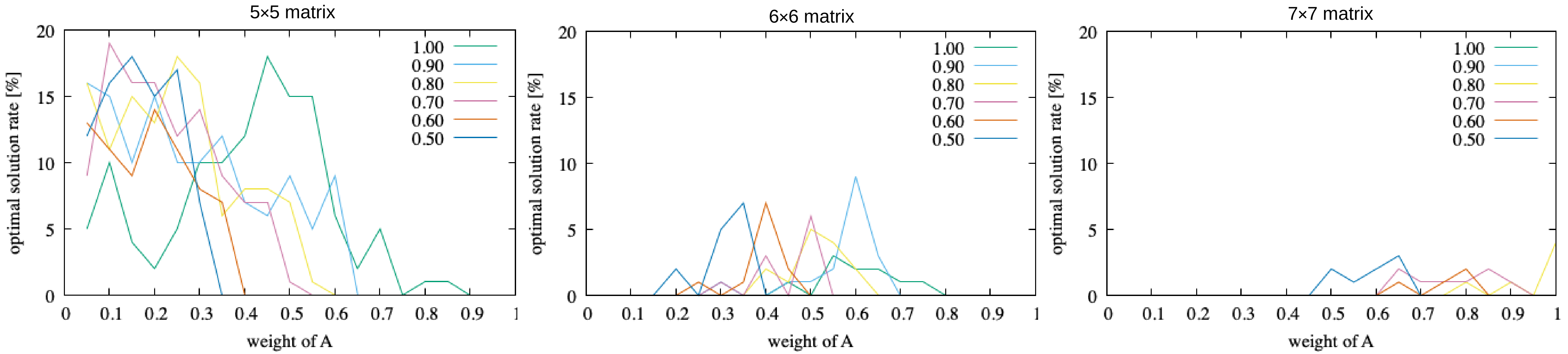
$$H = A * H_{cost} + B * H_{sub1} + C * H_{sub2}$$

➢ Fixed deletion order matrices

- used the matrices, which can be transformed into a cycle graph as fig.5
- Optimal solution **:** gives proper vertex deletion & adds specified edges



Fixed deletion order matrices



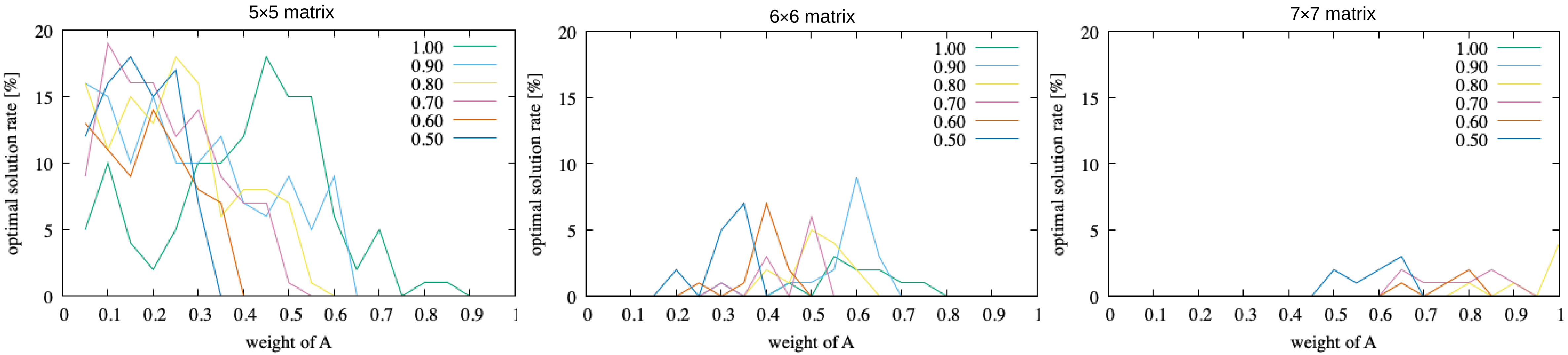
- The optimal solution rate decreases as the matrix size increases
- The optimal solution rate's peak also shifts toward larger values of A with matrix size or the value of C increases

→ the same parameters cannot be used for problems with different matrix sizes

→ parameter tuning is required to obtain the optimal solution

IV. Experimental result

➢ Fixed deletion order matrices



- These results also show that the parameters when the optimal solution is obtained are

  A<C for the 5×5 and 6×6matrices, but A ≥ C for the 7 × 7 matrix.

  → it is not enough to weight each function in the order of its priority

IV. Experimental result

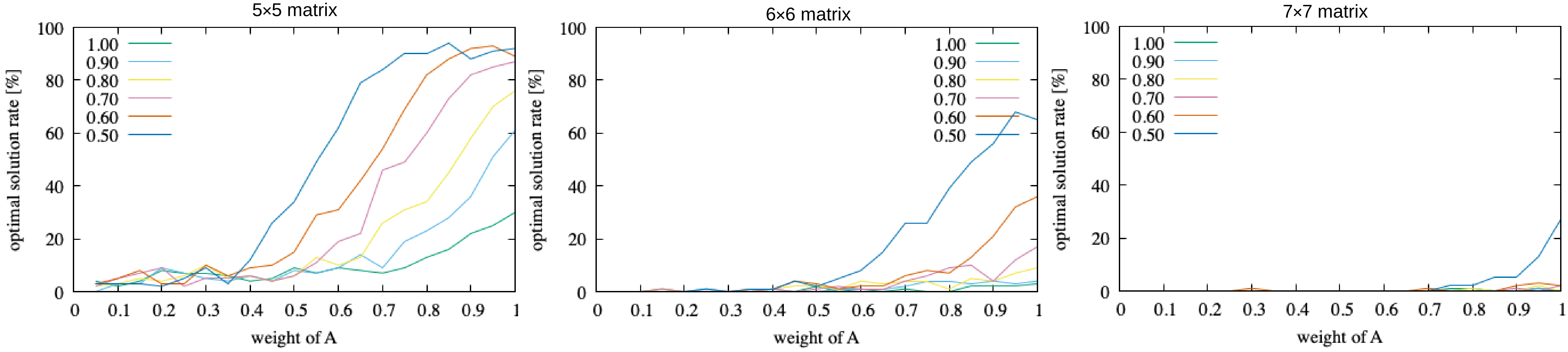
➢ **Random matrices**

- Used randomly generated matrices in which each element is nonzero with a probability of 1/3
- Evaluate: count the number of fill-ins of QMF, without ordering(Non) & MD
  - Optimal solution: “QMF ≤ MD” fill-ins
  - Local optimal solution: “MD < QMF ≤ Non” fill-ins

IV. Experimental result

- **Random matrices**

The optimal solution rate



- The solution rate changes with parameter tuning: 94% (5×5 matrices), 68% (6×6 matrices), 27% (7×7 matrices)
- The solution rate decreases as the matrix size increases
- The solution rate increases as the value of A is increased and decreases as the value of C is increased

→ giving high weights to the constraints we want to satisfy will not give an optimal solution

Outline

V. Conclusion

- The necessity and effectiveness of parameter tuning from the results of QMF ordering
  - Tuning weight parameters of QMF Hamiltonian is essential for the Ising machine to easily find the optimal solution
  - Parameter tuning can improve the rate at which Amplify AE finds an optimal solution by a maximum of 94% for 5 × 5 matrices, 68% for 6 × 6 matrices, 27% for 7 × 7 matrices
  - Giving high weights to the constraints we want to satisfy will not always give an optimal solution

- Parameter tuning for QA
  - There hasn't yet been enough discussion & it is being performed empirically
  - Exhaustive search takes long time
  - Auto-tuning is expected to decrease tuning time